April 6, 1994 WOCOMAL VARSITY MEET

ROUND I: Elementary number theory

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM

1. $x\overline{)y}$ 0 remainder 3

In this division problem, if $x>0$, which of the following must be true? Answer by letter.

A) $x=3$ B) $x=3y$ C) $y=0$ D) $y=3$ E) $y=3x$

2. This equation is written in base 5:

$$21x - 22 = 12x + 31$$

Solve it for x, and give the answer in base 5.

3. In base 10, how many "9" digits does this number have?

$$(10^{200} + 10^{20})(10^{200} - 10^{20})$$

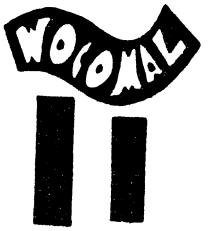
ANSWERS

(1 pt) 1. _____

(2 pts) 2. five

(3 pts) 3. _____

Hudson, Tantasqua, Worcester Academy



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ROUND II: Algebra 1 - open

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM

1. Simplify: $(\sqrt{12} + \sqrt{20})(\sqrt{3} - \sqrt{5})$

2. Simplify: $\dfrac{2^{n+4} - 2(2^n)}{2(2^{n+3})}$

3. Twice the result of subtracting 28 from five times a certain number is divided by the number decreased by 2 and the result is the same as that obtsined by subtracting 4 from the number. Find the number.

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

Auburn, Bancroft, Clinton

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ROUND III: Theory of polynomial equations and functions

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM

1. Write a quadratic equation in $ax^2+bx+c = 0$ form that has integer coefficients with no common factor >1 and a>0 and has roots $-1/2$ and 5.

2. Solve: $2ix^2 + 3x + 2i = 0$. Do not have a radical or an i in a denominator if any root is a fraction. ($i = \sqrt{-1}$)

3. If $x^3 - 3x^2 + kx + 75 = 0$ and one root is the additive inverse of another, find k.

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

Shepherd Hill, South, West Boylston

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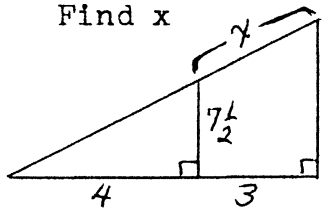
ROUND IV: Similarity and Pythagorean relationships

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM

1. Right triangles ABC and DEF are similar, with right angles A and D. If BC = 25, AC = 10, and EF = 60, find DF.

2. Find x



3. An isosecles triangle has a base of 12 and an area of 48. Find the area of the square having one side on the base and a vertex on each leg of the triangle.

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

Burncoat, Clinton, Mass. Academy

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ROUND V: Trigonometry - open

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM OR AS DIRECTED IN THE PROBLEM

1. Sheila is standing 500 feet away from the base of a building. Her eyes are 5 feet above the ground. From this point the top of the building makes a 54° angle with a line parallel to the ground. How tall is the building, to the nearest foot?

   $\sin 54^\circ = .8090$, $\cos 54^\circ = .5878$, $\tan 54^\circ = 1.3764$

2. If $\sin^2\theta + \sin^2 2\theta + \sin^2 3\theta + \sin^2 4\theta = 9/4$, evaluate $\cos^2\theta + \cos^2 2\theta + \cos^2 3\theta + \cos^2 4\theta$.

3. In $\triangle ABC$, the ratio $\sin A : \sin B : \sin C = 5 : 7 : 9$. Find the ratio $\cos A : \cos B$, as a reduced ratio of two integers.

ANSWERS

(1 pt) 1. _____ ft

(2 pts) 2. _____

(3 pts) 3. _____

Hudson, Quaboag, South, Tahanto

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TEAM ROUND: Topics of previous rounds and open 2 points each

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM OR AS DECIMALS ROUNDED TO FOUR DECIMAL PLACES AND ON THE SEPARATE TEAM ANSWER SHEET

1. Find three integers in arithmetic progression whose product is prime.

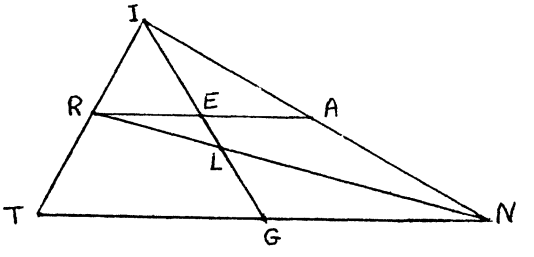
2. In a certain city a birth occurs on average every 24 minutes and a death every half hour. A resident moves out every 1.5 hours and a new person moves in every 4.5 hours. How long does it take on average for the population to increase by one person?

3. A cubic equation with integer coefficients has no quadratic term. If one root is $2+3i$, what are the other roots?

4. Let $\overline{AB}$ be a leg of the right triangle of least perimeter whose sides have integer lengths, whose hypotenuse is one unit longer than AB, and in which AB > 100. Find AB.

5. Find, in degrees, the negative angle θ closest to 0° for which $\log_2 \cos\theta = -1/2$.

6. If TR=RI, IA=AN, NG=GT, and IG=21, find EL.



7. A certain number of cactus plants cost \$9. Two more Venus fly-traps than that cost \$10. Ten cacti plus 4 Venus fly-traps cost \$20. What does one cactus cost?

8. What outside diameter of a hollow metal sphere, of inside diameter d, would be necessary in order that the hollow sphere, when melted down, form a solid sphere of diameter d?

9. The inverse function of $f(x) = 2x + |x|$ can be written in the form $f^{-1}(x) = \frac{ax + b|x|}{c}$ where a, b, and c are relatively prime integers. Find a+b+c.

Algonquin, Auburn, Bancroft, Bromfield, Quaboag, St.John's, St.Peter-Marian, South, Tahanto, Westboro

D

$(y=3$

12 five

360

-4

$\frac{7}{8}$ OR 0.875

8

$2x^2 - 9x - 5 = 0$

$2i, \frac{-i}{2}$

-25

24

$6\frac{3}{8}$ OR $\frac{51}{8}$ OR 6.375

$23\frac{1}{25}$ OR $\frac{576}{25}$ OR 23.04

693 ft

$\frac{7}{4}$ OR $1\frac{3}{4}$ OR 1.75

25:19 OR $\frac{25}{19}$

$-3, -1, 1$

18 hours OR 1080 min.
NEED UNITS

$2-3i, -4$

112

$-45°$

3.5

$1.50

$d\sqrt[3]{2}$ OR $1.2599d$

4